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# ANALYZING FIFO-MULTIPLEXING TANDEMS WITH NETWORK CALCULUS AND A TAILORED GRID SEARCH (SHORT PAPER)

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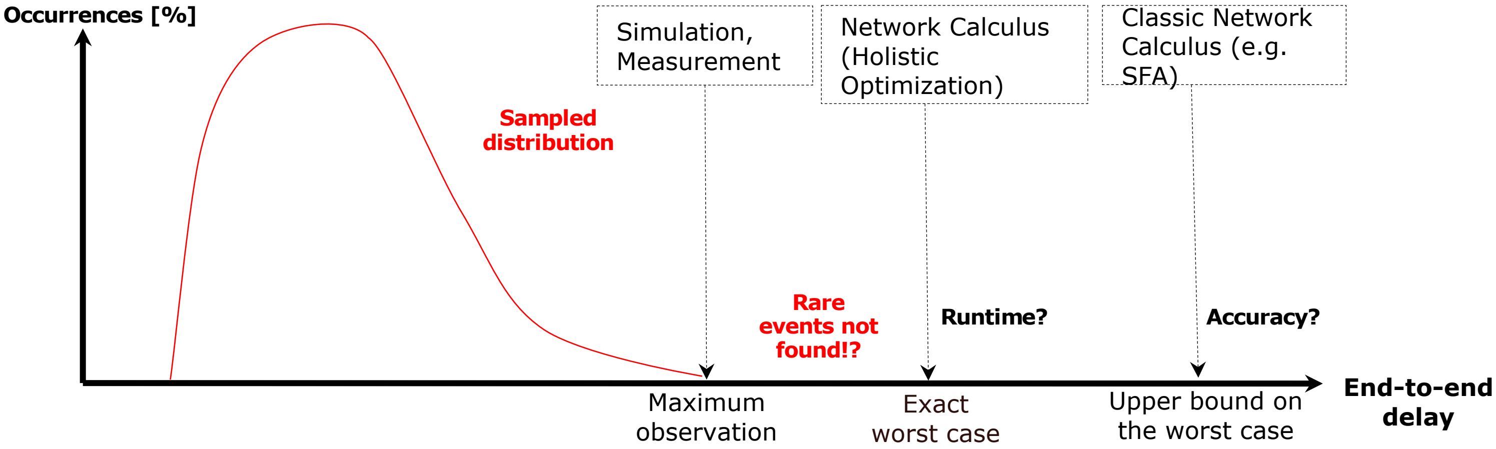
# Overview

- **Deterministic Network Calculus (DNC) Motivation and Basics**
- **Objective and Approaches**
- **Related Work**
- **GS**
- **Evaluation**

# DNC Motivation and Basics

- **Theory of deterministic queueing systems [Cruz91]**
  - Worst-case bounds such as delay and backlog

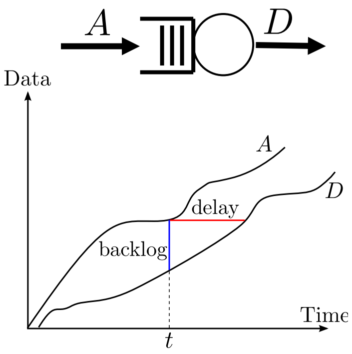


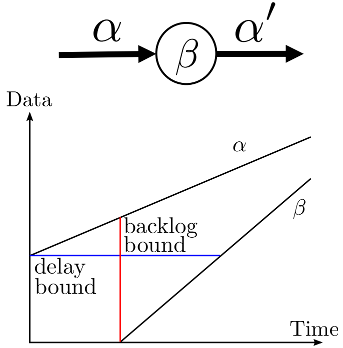
# DNC Motivation and Basics (2)

- **Can be used for certifying performance guarantees of cyber-physical systems, e.g., airplanes**
- **Can aid in ranking different network topologies and configurations**

# DNC Motivation and Basics (3) [LeBoudec01]

- **Arrival curve** $\forall 0 \leq s \leq t : A(t) - A(t-s) \leq \alpha(s)$
- **Service curve** $\forall t \geq 0 : A'(t) \geq \inf_{0 \leq s \leq t} \{A(t-s) + \beta(s)\} := A \otimes \beta(t)$



# DNC Motivation and Basics (4) [LeBoudec01]

- **Concatenation of servers** $\beta_1 \otimes \beta_2 = \beta_{1,2}$
- **Output bound** $\alpha'(t) = \alpha \oslash \beta(t) := \sup_{u \geq 0}\{\alpha(t+u) - \beta(u)\}$
- **Delay bound** $hdev(\alpha, \beta) = \inf\{d \geq 0 : (\alpha \oslash \beta)(-d) \leq 0\}$

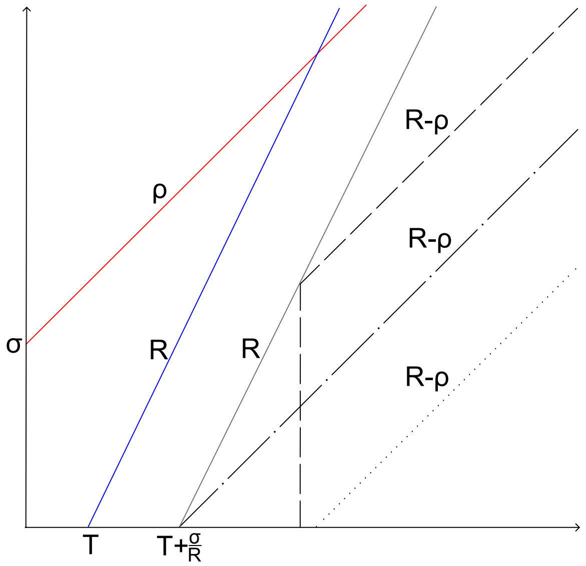
# DNC Motivation and Basics (5) [LeBoudec01]

- **FIFO left-over service curve**

$$\beta_{f_1}^{\text{l.o.}}(t,\theta) = [\beta(t) - \alpha_2(t-\theta)]^{\uparrow} \cdot 1_{\{t>\theta\}} \forall \theta \geq 0$$

$$[g(x)]^{\uparrow} = \sup_{0 \leq z \leq x} g(z)$$

$$1_{\{t>\theta\}} := \begin{cases} 1, & t > \theta \\ 0, & t \leq \theta \end{cases}$$

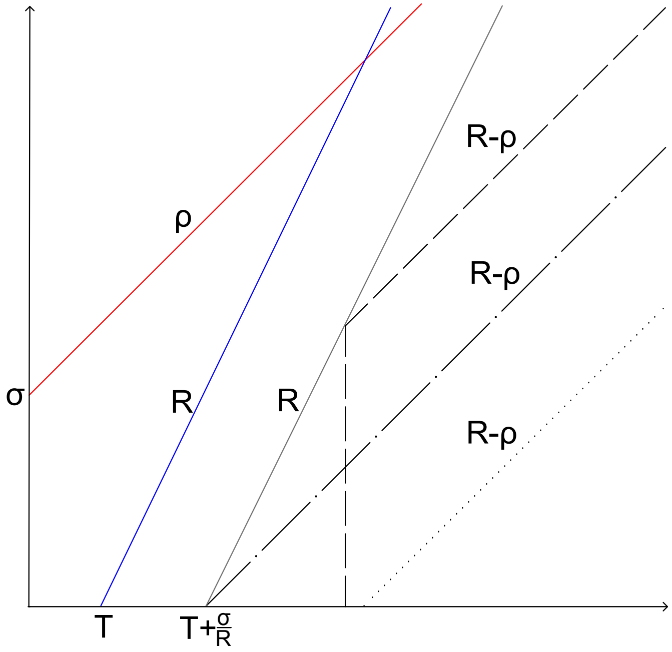


# Objective and Approaches

- **LUDB[Bisti08], LUDB-FF [Scheffler21] optimizes these free parameters**
- **New Approach**
  - Instead of optimizing these free parameters, we employ a robust grid search (GS) for a better tradeoff between accuracy and runtime
  - We use GS to rank different network topologies
- **Find our code and dataset at**
  - https://github.com/NetCal/DNC
  - https://github.com/alexscheffler/dataset-itc2022

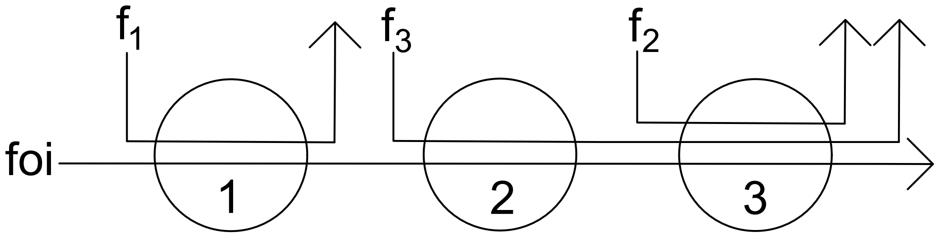
# Related Work

- **SFA-FIFO**
  - Server-by-server analysis
  - Compute at each server the residual service curve, convolve them
  - Each occuring $\theta$ set statically, $T + \frac{\sigma}{R}$
  - Simple left-over curve but local view



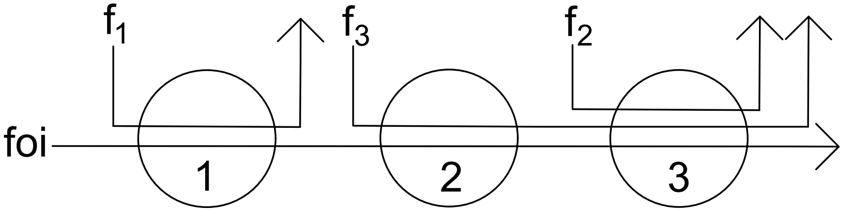
# Related Work (2)

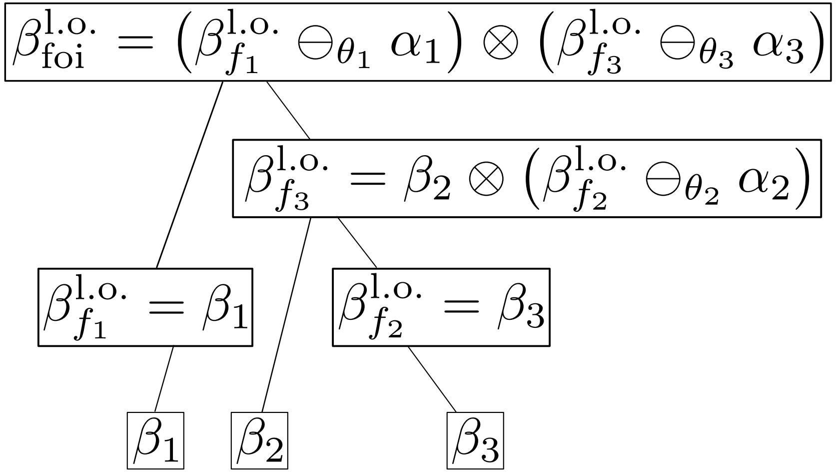
- **LUDB [Bisti08]**
  - Nested interference: A tandem has nested interference iff for every pair of flows either both flows do not have common servers or the path of one flow is completely included in the path of the other.



# Related Work (3)

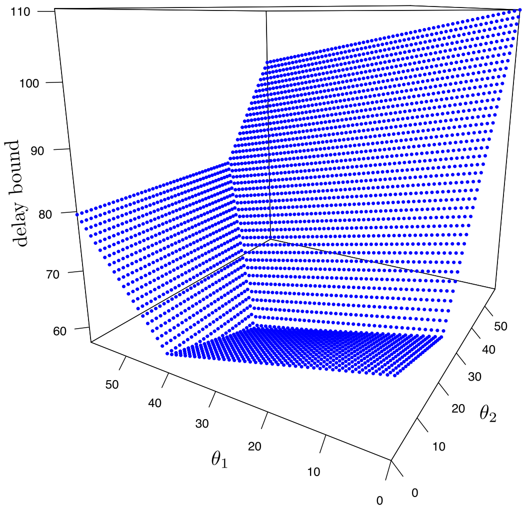
- **LUDB [Bisti08]**
  - "convolution before subtraction" for nested interference
  - Optimizes $(\theta_1, ..., \theta_{|F_x|})$ w.r.t. delay bound



# GS

- **Start with** $\Theta = (\theta_1, ..., \theta_{|F_x|}) = (0, ..., 0)$ **resulting in delay bound** $d^{\text{start}}$





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# GS (2)

- **Start with** $\Theta = (\theta_1, ..., \theta_{|F_x|}) = (0, ..., 0)$ **resulting in delay bound** $d^{\text{start}}$
- **Procedure**
  - Partition the search space: $|F_x|$- dimensional grid
  - Each point on the grid: delay bound $d$
  - Try, for each $\theta_i$, $g$ different values between $0$ and $d^{\text{start}}$ with $g \in \mathbb{N}_{>1}$
    - Stepsize: $sp := \frac{d^{\text{start}}}{g-1}$
    - Hence, $\theta_i \in \left[0, \frac{1}{g-1} d^{start}, \frac{2}{g-1} d^{start}, ..., \frac{g-1}{g-1} d^{start}\right]$



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# GS (3)

- **Procedure**
  - Results in $\mathcal{O}(g^{|F_x|})$, i.e., exponential in the number of crossflows
  - Check for each $\theta_i$, if $\theta_i \leq d^{\text{current}}$ holds, otherwise such a combination can be safely skipped

**Algorithm 2** Grid search on a nested tandem

**Input** $i, \Theta$ Flow, parameter combination
**Output** $\beta^{\text{l.o.}}$ Left-over service curve for flow $i$

```
procedure GS(i, Θ)
    for (θ_i ← 0; θ_i ≤ d^current; θ_i ← θ_i + sp) do
        Θ(i) ← θ_i
        if i == |F_x| then
            β^l.o._foi ← COMPUTELEFTOVERSERVICE(foi, Θ)
            d ← hdev(α_foi, β^l.o._foi)        ▷ horizontal deviation
            if d < d^current then
                d^current ← d
                β^l.o. ← β^l.o._foi
        else
            GS(i + 1, Θ)
    return β^l.o.
```



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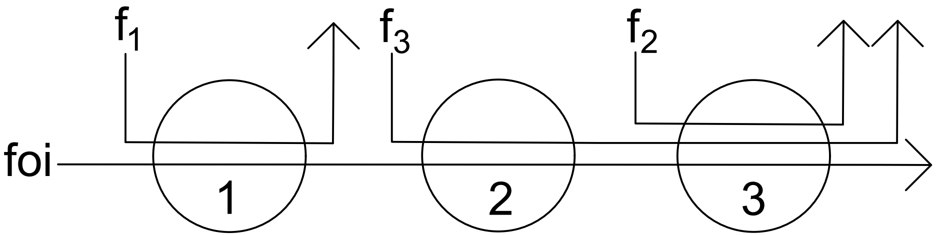
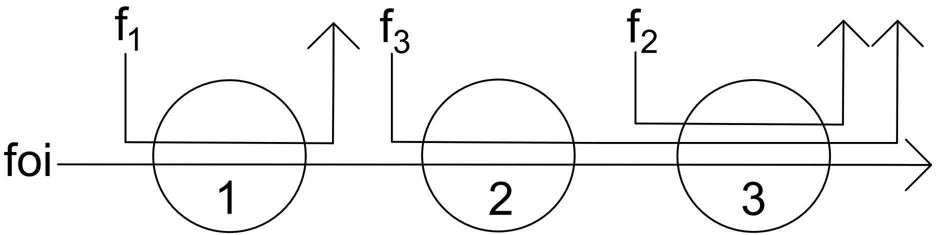
# GS (4)




- $|F_x| = 3$ , $g = 4$, $\theta_i \in \left[0, \frac{1}{3}d^{start}, \frac{2}{3}d^{start}, d^{start}\right]$
- **1st iteration:** $\Theta = (\theta_1, \theta_2, \theta_3) = (0, 0, 0) \rightarrow d = d^{\text{start}} = d^{\text{current}}$
- **2nd iteration:** $\Theta = (\theta_1, \theta_2, \theta_3) = (0, 0, \frac{1}{3}d^{\text{start}}) \rightarrow d$ **, update** $d^{\text{current}}$ **if** $d < d^{\text{current}}$
- **3rd iteration:** $\Theta = (\theta_1, \theta_2, \theta_3) = (0, 0, \frac{2}{3}d^{\text{start}}) \rightarrow d$**, update** $d^{\text{current}}$ **if** $d < d^{\text{current}}$
- **4th iteration:** $\Theta = (\theta_1, \theta_2, \theta_3) = (0, 0, d^{\text{start}}) \rightarrow d$**, update** $d^{\text{current}}$ **if** $d < d^{\text{current}}$

# GS (5)




- $|F_x| = 3$ **,** $g = 4$**,** $\theta_i \in \left[0, \frac{1}{3}d^{start}, \frac{2}{3}d^{start}, d^{start}\right]$
- **4th iteration:** $\Theta = (\theta_1, \theta_2, \theta_3) = (0, 0, d^{\text{start}}) \rightarrow d$ **, update** $d^{\text{current}}$ **if** $d < d^{\text{current}}$
- **5th iteration:** $\Theta = (\theta_1, \theta_2, \theta_3) = (0, \frac{1}{3}d^{\text{start}}, 0) \rightarrow d$**, update** $d^{\text{current}}$ **if** $d < d^{\text{current}}$
- **...**
- **64th iteration:** $\Theta = (\theta_1, \theta_2, \theta_3) = (d^{\text{start}}, d^{\text{start}}, d^{\text{start}}) \rightarrow d$**, update** $d^{\text{current}}$ **if** $d < d^{\text{current}}$

# GS (6)

- **Lemma 1: Let $g_1, g_2 \in \mathbb{N}_{>1}$ with $k \cdot (g_1 - 1) = g_2 - 1$, for $k \in \mathbb{N}$. Then,**

  $\mathrm{delay}(\text{GS-}g_1) \geq \mathrm{delay}(\text{GS-}g_2)$ **holds.**

- **Proof:**
  - GS-$g_1$ : Each $\theta_i \in \left[0, \frac{1}{g_1 - 1} d^{start}, \frac{2}{g_1 - 1} d^{start}, \ldots, \frac{g_1 - 1}{g_1 - 1} d^{start}\right]$
  - GS-$g_2$: Each $\theta_i \in \left[0, \frac{1}{g_2 - 1} d^{start}, \frac{2}{g_2 - 1} d^{start}, \ldots, \frac{g_2 - 1}{g_2 - 1} d^{start}\right]$

    $$\overset{k \cdot (g_1 - 1) = g_2 - 1}{=} \left[0, \frac{1}{k} \frac{d^{\text{start}}}{g_1 - 1}, \frac{2}{k} \frac{d^{\text{start}}}{g_1 - 1}, \ldots, \frac{k \cdot (g_1 - 1)}{k} \frac{d^{\text{start}}}{g_1 - 1}\right]$$

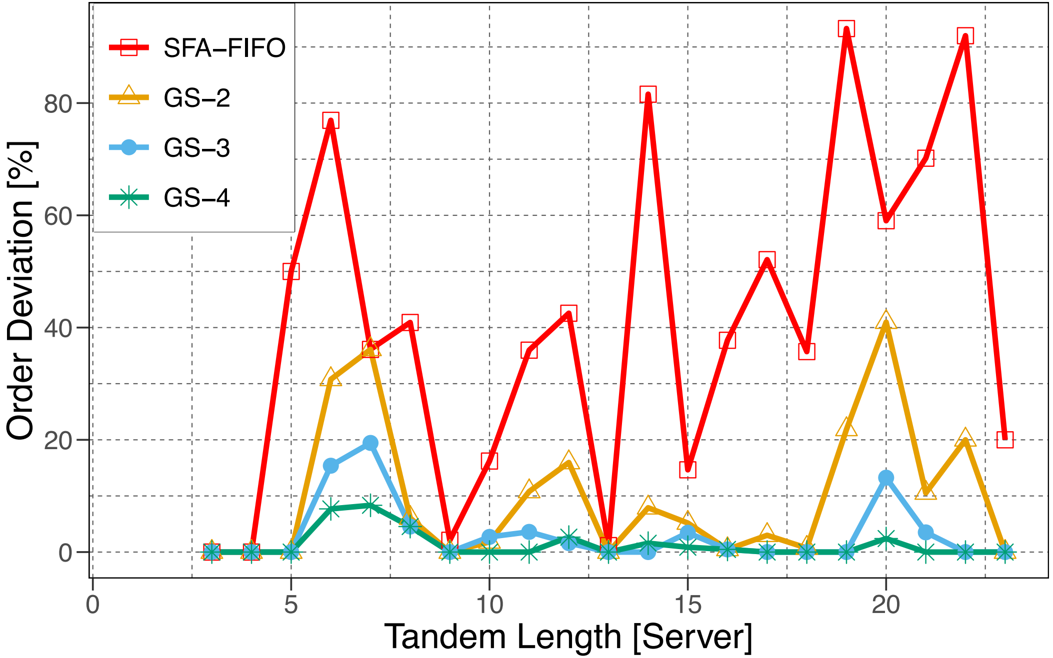
  - $\theta_i(\text{GS-}g_1) = \frac{a}{g_1 - 1} d^{\text{start}} \Rightarrow \theta_i(\text{GS-}g_2) = \frac{b}{g_2 - 1} d^{\text{start}}$ with $a \in \mathbb{N} \cap [0, g_1 - 1], b = a \cdot k \in \mathbb{N} \cap [0, g_2 - 1]$

# Evaluation

- **Setup**
  - 2086 unique nested tandems
  - Arrival curves set to token bucket $\gamma_{\rho,\sigma} = \gamma_{1,1}$
  - Service curve set to rate latency $\beta_{R,T}$ with $T = 0$ and $R$ set to achieve a utilization of each server of 95%
- **Goal**
  - Find reasonable value of g (GS parameter) s.t. the ranking of the different tandems is close to the ranking with the LUDB analysis

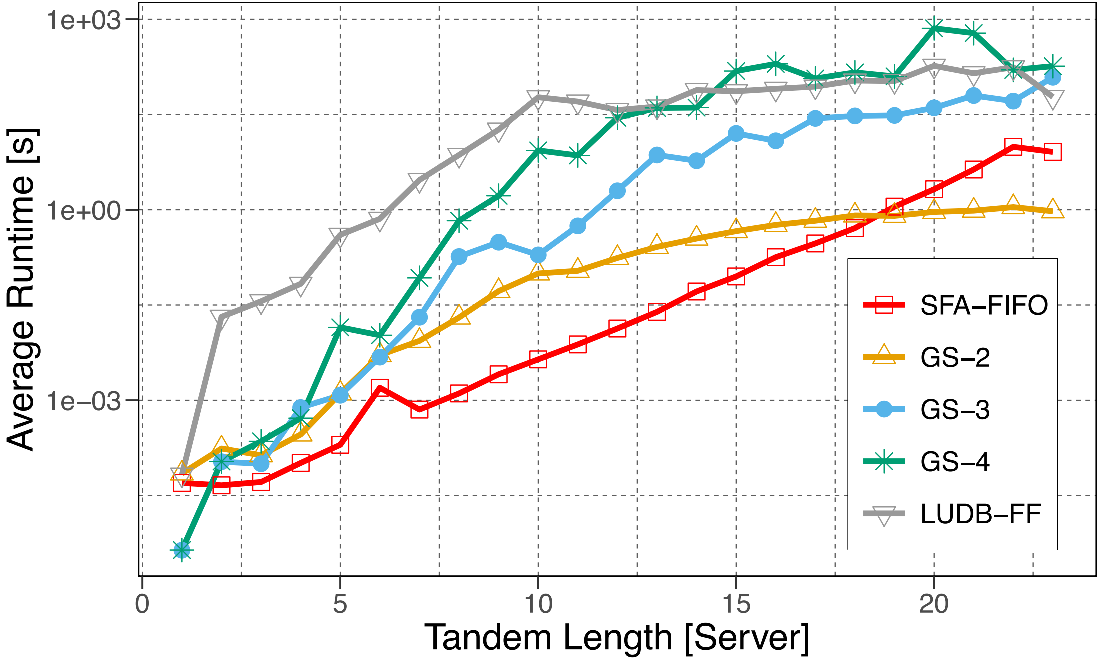
# Evaluation (2)

- **Ranking deviation compared to LUDB**



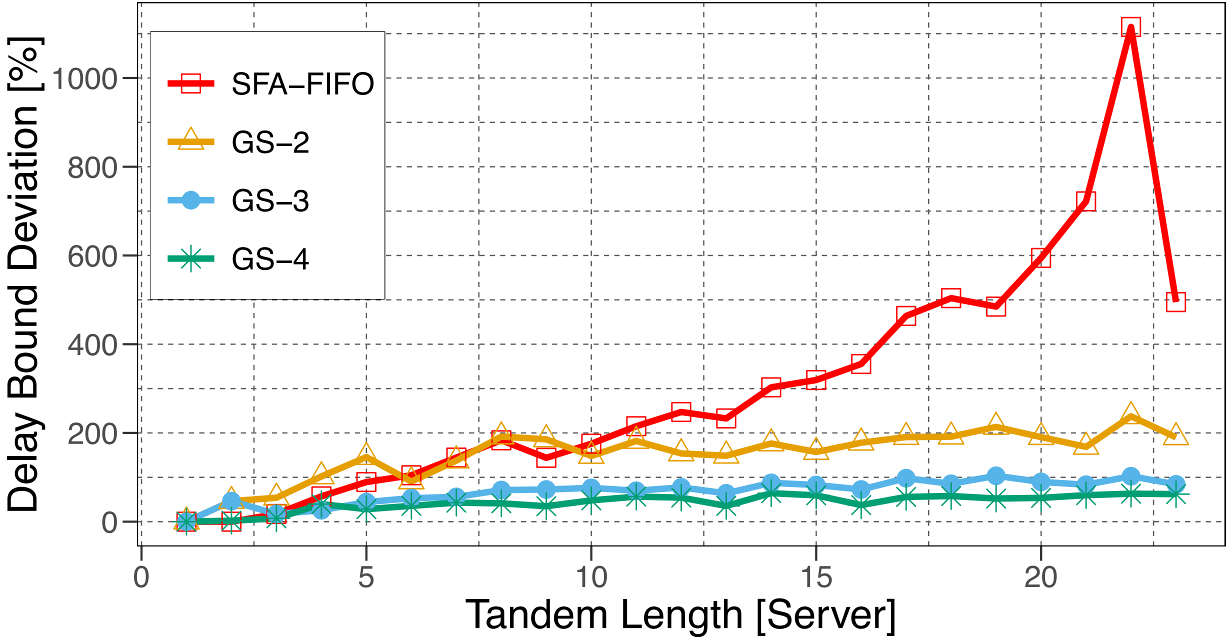
# Evaluation (3)

- **Average analysis runtime**



# Evaluation (4)

- **Delay bound deviation of the top-ranked tandems**



# Conclusion

- **Exploration of network design space w.r.t. delay bounds in FIFO-Multiplexing tandems**
- **Benchmark several NC analyses for this task**
  - SFA-FIFO's bounds too coarse: Ranking deviation of up to 93% compared to LUDB-FF
  - New analysis GS:
    - Worst-Case order deviation of no more than 41% while GS is considerably faster than LUDB-FF
    - Precision and runtime can be improved with a parameter
    - g of 2-4 most suitable for striking a reasonable balance between quality of ranking and runtime

**Thanks for your attention!**

**Questions?**

# References

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*[Scheffler21] Alexander Scheffler and Steffen Bondorf. 2021. Network Calculus for Bounding Delays in Feedforward Networks of FIFO Queueing Systems. In Proc. of the International Conference on Quantitative Evaluation of Systems (QEST).*